[9]Gurdal, Z., and Olmedo, R., "In-Plane Response of Laminates with Spatially Varying Fiber Orientations: 'Variable Stiffness Panel Concept'," *AIAA Journal*, Vol. 31, No. 4, 1993, pp. 751–758.

[10]Biggers, S. B., and Srinivasan, S., "Compression Buckling Response of Tailored Rectangular Composite Plates," *AIAA Journal*, Vol. 31, No. 3, 1993, pp. 590–596.

[11]Biggers, S. B., and Srinivasan, S., "Improved Compression Buckling for Rectangular Composite Plates by Stiffness Tailoring," *Enhancing Analysis Techniques for Composite Materials*, NDE Vol. 10, ASME, New York, 1991, pp. 187–195.

[12]HKS (Hibbitt, Karlsson, and Sorensen), Inc., *ABAQUS Theory and User's Manuals, Version 4.8*, HKS, Pawtucket, RI, 1989, pp. 2.2.7-1, 3.4.3-1–3.4.3-6.

[13]Dow, M. B., and Smith, D. L., "Properties of Two Composite Materials Made of Toughened Epoxy Resin and High-Strain Graphite Fiber," NASA TP-2826, 1988.

Damage Detection in Framed Structures

Izhak Sheinman*
Technion—Israel Institute of Technology, Haifa, Israel

Introduction

INTRODUCTION of advanced new materials and "smart" structures is strongly conditional on ability to assure their safety. In the course of their service life, load-carrying structural systems undergo damage which should be monitored with respect to occurrence, location, and extent. The new nondestructive test procedures developed following the rapid advances in testing equipment and techniques are mostly based on modal analysis and entail a relatively large volume of measurements which limits their suitability for practical application. Hence the need for a solution involving minimal measurements.

Systematic methods put forward in recent years and combining test data with analysis may be used for damage detection as well. Some of these (for example, Refs. 1 and 2) propose optimization procedures for correcting the stiffness and mass matrix on the basis of measured modes, irrespective of the physical meaning of the connectivity. Others (like Refs. 3 and 4) offer mathematical procedures which preserve the original connectivity and consequently reduce the required volume of measurements. A closed-form algorithm for precise detection, using test data and likewise preserving the connectivity, is given in Ref. 5. This algorithm identifies the damaged degrees of freedom (DOF) and then solves a set of equations to yield the damaged stiffness coefficients. Its drawback is that even a small number of damaged DOFs may result in a large number of damaged stiffness coefficients with the corresponding excessive measurement volume. Accordingly, this Note presents a new algorithm which preserves the ratio of stiffness coefficients besides the connectivity, and thus significantly reduces the needed measurements. The algorithm identifies the damaged members through very few measured modes, and is suitable for large structures with thousands of degrees of freedom. It is suitable for framed structures in which the DOFs are disturbed by the damage to the members rather than by that to the joints, so that the algorithm should refer to the affected members instead of the stiffness coefficients.

The algorithm is developed for the general case of damage in both the stiffness and mass matrices, in which case static measurements are for the former, and then dynamic ones for the latter. Where only one matrix is damaged, either static or dynamic measurements are called for in the case of the stiffness matrix, and only dynamic ones in that of the mass matrix. The present algorithm is confined for framed structures (truss and beam members), and does not address the issue of noise, as it is based on real test data (which are assumed to be accurate).

Received May 4, 1993; revision received Sept. 29, 1993; accepted for publication Sept. 30, 1993.

*Professor, Faculty of Civil Engineering.

Analytical Formulation

The procedure is based on the following equation:

$$[\Delta R]\{\phi_{im}\} = \{y_{di}\} \tag{1}$$

where

$$[\Delta R] = [\Delta K] - \omega_{im}^2[\Delta M]$$
$$\{y_{di}\} = [\omega_{im}^2 M_o - K_o]\{\phi_{im}\} \tag{2}$$

K_o and M_o are the stiffness and mass matrices of the undamaged system; ΔK and ΔM are the respective amounts of damage to the matrices; ω_{im} and ϕ_{im} are the measured ith frequency and ith eigenmode; and y_{di} is the force residue at all DOFs created by the damage.

The location of the damage is determined directly by identifying the DOFs (j) whose force residue $y_{di}(j)$ is not zero. This can be done through a single measured mode, provided the latter is affected by the DOF in question. Since ΔR contains the unknown stiffness and mass coefficients, it is desirable to rewrite Eq. (1) as

$$[C]\{\bar{\Delta R}\} = \{Z_{di}\} \tag{3}$$

where $\bar{\Delta R}$ is the unknown vector containing only the terms of ΔRej appearing in the jth equation for which $y_{di}(j) \neq 0$. Z_{di} is the vector consisting of the nonzero terms of y_{di} and $[C]$ the coefficient matrix consisting of the measured eigenmode parameters.

As the connectivity is assumed to be preserved, $[C]$ can be decomposed into uncoupled regions (of order N, N being the number of unknown stiffness coefficients for the current damaged zone) which are solvable separately (see Ref. 5), with the attendant significant saving in computer time. It should be noted that Eq. (3) yields the exact $\bar{\Delta R}$ vector provided the measured modes are exact. However, the number of measured modes which are needed for solving Eq. (3) is increasing with the number of DOFs of the ends of the damage zone. For example, for a plane beam element it may require up to four measured modes and for a space beam element up to 12 measured modes (irrespective of the number of elements in the damaged zone).

In the case of a damaged stiffness matrix, a procedure based on preserving the ratio of stiffness coefficients (meaning the effect of damage is the same on all terms which are connected) makes for significant reductions of the number of measured modes. Preservation of the connectivity, irrespective of where the modes are measured, yields the exact damage vector $\bar{\Delta R}$. By contrast, preservation of the ratio of stiffness coefficients implies that the damage is uniquely distributed in between the measured damaged DOFs. Hence, for the ratio to be preserved the damaged zone should be exactly located, the modes are then measured at its ends and the damage of the stiffness matrix, at each affected zone, is calculated from

$$[C]_{N*N}\{\Delta\bar{K}\}_{N*1} = \{Z_{di}\}_{N*1} \tag{4}$$

(in the case where the mass matrix is also damaged, Eq. (4) is derived from static measurements).

Since the coefficient ratios of the stiffness matrix are preserved, $\{\Delta\bar{K}\}$ can be expressed in terms of the damaged properties of the members. In framed structures, these are EA, GJ, EI_1, and EI_2, where E is the modulus of elasticity, G is the shear modulus, A is the cross-sectional area, and J, I_1, I_2 are the moments of inertia in the axial and transverse directions. For general case of a different damage amount to each property, the local stiffness matrix of the member (which comprised the submatrix for the jth end in the member stiffness matrix) can be written as

$$[K]_m = (EA)_m[K_1]_m + (GJ)_m[K_2]_m + (EI_1)_m[K_3]_m + (EI_2)_m[K_4]_m \tag{5}$$

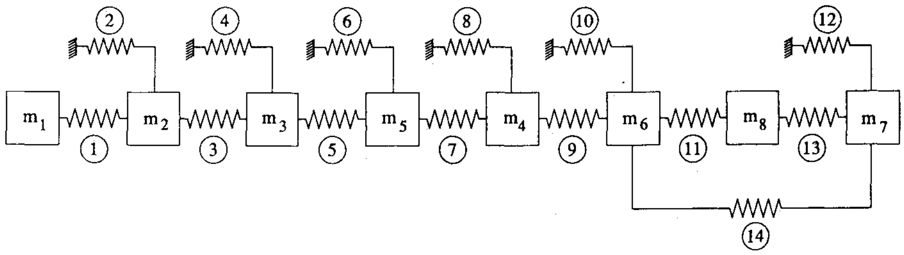

Fig. 1 Kabe's (Ref. 3) mass-spring system.

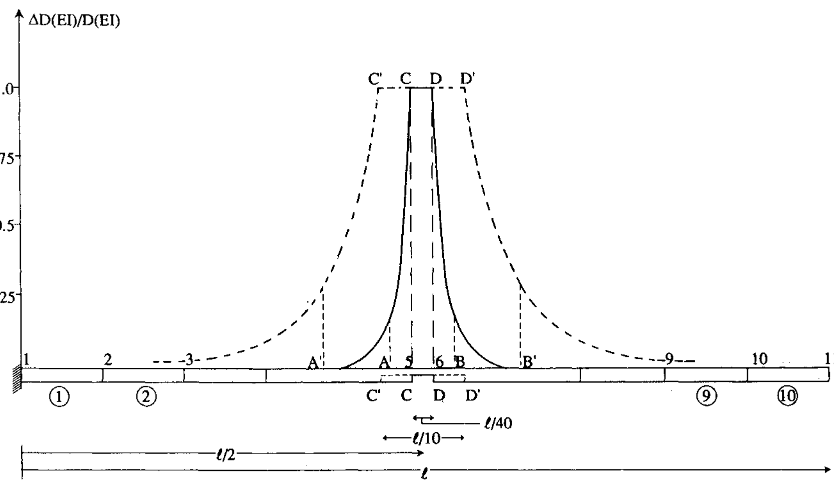

Fig. 2 Damaged cantilever beam.

where $[K_i]_m$ are the member (m) submatrices whose nonzero terms are (see Ref. 6)

$$K_1(1, 1) = K_2(4, 4) = 1/\ell \tag{6a}$$

$$K_3(2, 2) = K_4(3, 3) = 12/\ell^3 \tag{6b}$$

$$K_3(2, 6) = K_3(6, 2) = -K_4(3, 5) = -K_4(5, 3) = 6/\ell^2 \tag{6c}$$

$$K_3(6, 6) = K_4(5, 5) = 4/\ell \tag{6d}$$

where ℓ is the length of the damaged member.

Next, the unknown stiffness vector can be expressed in terms of the member properties as

$$\{\Delta \bar{K}\}_{N*1} = [\psi]_{N*M}\{\Delta E\}_{M*1} \tag{7}$$

where $\{\Delta E\}$ comprises the four properties $(EA)_m$, $(GJ)_m$, $(EI_1)_m$, and $(EI_2)_m$ for each member in the current damaged zone, of order $M = 4L$ (L being the number of members in the zone); and $[\psi]$ is the coefficient matrix comprising the stiffness ratio given in Eq. (6). Actually, this matrix expresses preservation of the ratios of the stiffness coefficients.

Substitution of Eq. (7) in Eq. (4) yields

$$[CE]_{N*M} \cdot \{\Delta E\}_{M*1} = \{Z_{di}\}_{N*1} \tag{8}$$

where

$$[CE]_{N*M} = [C]_{N*N} * [\psi]_{N*M} \tag{9}$$

The number of unknowns in Eq. (8) is thus reduced to M while the number of equations remains N ($N \gg M$). Where the measurements are taken exactly at the ends of the damaged member, the additional equations ($M+1, M+2, \ldots, N$) are combinations of the first M equations and only these need be considered. Otherwise, the additional equations conflict with the first M equations and an exact solution does not exist; but even then the first M equations can give an indication of the damage. Provided the mode is affected by all DOFs, the damage can be exactly located through a single measured mode.

Illustrative Examples

To illustrate the procedure and the efficiency of the method based on preserving the stiffness ratio, two examples are provided. The first example, reproduced from Ref. 3, is the string mass system shown in Fig. 1. It contains eight DOFs and 14 spring elements. If all elements are damaged as in Ref. 3, the number of damaged DOFs is eight and preservation of the connectivity yields 16 unknown stiffness coefficients. Since each mode contributes eight equations, so that the number of needed measured modes is at least two—but since the rank of the coefficient matrix [Eq. (3)] is 15—smaller than its order, the actual number is three (eight equations of the first mode, seven of the second mode, and one of the third). By contrast, preservation of the ratios of stiffness coefficients results in 14 unknowns (EA of each spring element) and only two measured modes are needed.

If only part of the spring elements are damaged, the number may, in certain cases, be even smaller. For instance, when the damaged element is element no. 1, the first two DOFs are damaged with three unknowns [$\Delta K(1, 1)$, $\Delta K(1, 2)$ and $\Delta K(2, 2)$], since in this case, each mode contributes only two equations (the number of corrupted DOFs), two measured mode are needed, but if the stiffness ratio is preserved there is only one unknown (EA of element no. 1) and a single measured mode suffices.

The second example deals with a plane cantilever beam represented by ten elements with two DOFs (translation and rotation) at each nodal point (a total of 20 DOFs). Two different cases of a distinct damaged zone located at midspan (see Fig. 2)—C-D of $\ell/40$ and C′-D′ of $\ell/10$—are considered. The first step involving location of the damaged zone C-D (or C′-D′) by using the first measured mode and by trial and error, through shifting of the measured nodal points 5 and 6 (see Fig. 2), is carried out with reference to the residue y_{di} [Eq. (2)]. Once the damaged zone has been exactly located (represented by DOFs 7, 8, 9, and 10), the extent of the damage can be sought. Preservation of the connectivity yields ten unknowns for the stiffness coefficients [$\Delta K(7, 7)$, $\Delta K(7, 8)$, ..., $\Delta K(10, 10)$]. Since each measured mode contributes four equations, at least three of them are required. However, since the modes are not necessarily mutually orthogonal over the damaged zone, it happens that the rank of the coefficient matrix $[C]$ is only 9 (while its order is 10) and a fourth measured mode has to be considered for the exact damage extent. It should be noted that under preservation of the connectivity alone even if nodal points 5 and 6 (see Fig. 2) are not at the exact ends of the damaged zone, the stiffness [$\Delta K(i, j)$] are determined exactly through Eq. (4) (using four measured modes), but it is difficult to define the character of the damage from these exact coefficients. Preservation of the stiffness ratio besides the connectivity requires that the measured nodal points coincide with the exact ends of the damaged zone. This yields a single unknown (EI) and therefore only one measured mode is needed. When the ends are not included, the stiffness ratio is not preserved, but can be used as an indication as illustrated in Fig. 2. In this figure, C-D (or C′-D′) is the real damaged zone and A and B (or A′-B′) are the location of the measured nodal points (no other point in between), $D(EI)$ is the given extent of the damage to the beam rigidity EI (80% in the present case); and $\Delta D(EI)$ is the extent calculated from the measured nodal points according to the location of A and B (A′-B′). Apparently, when A and B are located exactly at the ends (point 5 and 6), the exact extent is obtainable [$\Delta D(EI)/D(EI) = 1$]. The further the nodal points are away from the ends, the lower the predicted damage extent. The results are shown for both different damaged zones $\ell/40$ (solid line) and $\ell/10$ (dashed line). It was also checked for different boundary conditions and the results were the same indicating that it is a local phenomenon.

Concluding Remarks

Preservation of the connectivity of the stiffness and mass matrices yields a procedure for exact damage detection and location, but the required number of measured modes can be prohibitive for practical problems. By contrast, under preservation of the stiffness ratio (which is perfectly reasonable when the measurements include the ends of the damaged zone) no more than one or two measured modes (which are affected by all DOFs) are usually needed. Since the location is characterized by a single mode, the exact ends can be included in it. The proposed procedure is highly promising especially for large structures.

References

[1]Baruch, M., "Optimization Procedure to Correct Stiffness and Flexibility Matrices Using Vibration Test," *AIAA Journal*, Vol. 16, No. 11, 1978, pp. 1208–1210.

[2]Berman, A., and Nagy, E. J., "Improvement of a Large Analytical Model Using Test Data," *AIAA Journal*, Vol. 21, No. 8, 1983, pp. 1168–1173.

[3]Kabe, A. M., "Stiffness Matrix Adjustment Using Mode Data," *AIAA Journal*, Vol. 23, No. 9, 1985, pp. 1431–1436.

[4]Kammer, D. C., "Optimal Approximation for Residual Stiffness in Linear System Identification," *AIAA Journal*, Vol. 26, No. 1, 1988, pp. 104–112.

[5]Sheinman, I., "Damage Detection and Updating of Stiffness and Mass Matrices with the Aid of Test Data," *Journal of Modal Analysis* (submitted for publication).

[6]Gere, J. M., and Weaver, W., Jr., "Analysis of Framed Structures," Van Nostrand Co., New York, 1965.